2021

MATHEMATICS — HONOURS

Paper : CC-14

(Numerical Methods)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Each of the following questions has four possible answers of which exactly one is correct. Choose the correct alternative : 1×10

(a) The second degree polynomial passes through (0, 5), (2, 7), (4, 17), (6, 35) is

(i) $x^2 - 3x + 5$ (ii) $3x^2 - x - 5$

(iii) $x^2 - x + 5$ (iv) $x^2 + x - 5$.

(b) Which of the following is a suitable fixed point iteration for solving the equation $\cos(x) - x = 0$?

(I) $x_{n+1} = \cos(x_n)$ (II) $x_{n+1} = \cos^{-1}(x_n)$ (III) $x_{n+1} = x_n + \tan(x_n)$

(i) I (ii) I and II

(iii) I and III (iv) All of these.

(c) The process of constructing a sequence of vectors and obtaining the solution of a system using specified accuracy is called

(i) Elimination (ii) Reduction

(iii) Iteration (iv) Raphson method.

(d) Let $f(x) = x^5 + x^4 - 3$. How many steps of the bisection method are required to approximate the root to within 10^{-100}?

(i) 537 (ii) 333

(iii) 143 (iv) 153.

(e) If $f(x)$ has an isolated zero of multiplicity 3 at $x = \xi$, and the iteration $x_{n+1} = x_n - \dfrac{3f(x_n)}{f'(x_n)}$ converges to ξ, then the rate of convergence is

(i) linear (ii) faster than linear but slower than quadratic

(iii) quadratic (iv) cubic.

(f) If e_x be the relative error in the computation of the approximate value of $X(X > 0)$, the relative error while computing the value of $X^{\frac{1}{10}}$ is

(i) $e_x^{\frac{1}{10}}$ (ii) $10e_x$

(iii) $\frac{e_x}{10}$ (iv) also e_x.

(g) The two-point central difference formula for $f'(x)(h > 0)$ is

(i) $\frac{f(x+h)-f(x-h)}{h}$ (ii) $\frac{f(x+h)-f(x-h)}{2h}$

(iii) $\frac{f\left(x+\frac{h}{2}\right)-f\left(x-\frac{h}{2}\right)}{2h}$ (iv) $\frac{f(x+h)-f(x)}{h}$.

(h) The sum of Cote's co-efficients of Simpson's $\frac{3}{8}$ rule for evaluating $\int_1^{10} f(x)dx$ is

(i) $\frac{3}{8}$ (ii) $\frac{8}{3}$

(iii) 1 (iv) 9.

(i) If we consider $\frac{dy}{dx} = f(x)$ for $y(x_0) = y_0$, Simpson's $\frac{1}{3}$rd rule is equivalent to

(i) Euler's method (ii) second-order Runge–Kutta method

(iii) fourth-order Runge–Kutta method (iv) Modified Euler's method.

(j) The Gauss–Jacobi's iteration method for the set of equations

$$x_1 + 3ax_2 = 7,\ ax_1 + x_2 = 4, \left(a \neq \frac{1}{\sqrt{3}}\right) a \neq -\frac{1}{\sqrt{3}}$$

will surely converge for

(i) all values of a (ii) $a = 1$

(iii) $|a| < \frac{1}{\sqrt{3}}$ (iv) $\frac{1}{\sqrt{3}} < a < \frac{2}{\sqrt{3}}$.

Unit - 1 and Unit - 2

Answer ***any one*** question.

2. (a) Prove that $f'(x)=\frac{1}{h}\left[\Delta-\frac{\Delta^2}{2}+\frac{\Delta^3}{3}-\frac{\Delta^4}{4}+....\right]f(x)$. (*The symbols have usual meaning*).

(b) Find the number of significant figures in the approximate number 1.8921, given its relative error is 0.1×10^{-2}. 3+2

3. Find the number of terms of the exponential series such that their sum gives the value of e^x correct to six decimal places at $x = 1$. What are the basic features of Hermite interpolation formula? 2+3

Unit - 3

Answer ***any two*** questions.

4. (a) The quadrature formula

$$\int_0^1 f(x)dx=\frac{1}{2}f(x_0)+c_1f(x_1)$$

has the highest degree of precision. Determine x_0, c_1 and x_1.

(b) Correct or justify : Simpson's $\frac{3}{8}$ rule is slightly more accurate than Simpson's $\frac{1}{3}$ rule. 3+2

5. (a) Established the midpoint rule

$$\int_{x_0}^{x_1} f(x)dx=hf\left(x_0+\frac{h}{2}\right)+\frac{h^3}{24}f''(\xi),\ x_0\le\xi\le x_1,\ h=\frac{x_1-x_0}{2}.$$

(b) Correct or justify : For each integer $n > 3$, there exists a polynomial $P_n(x)$ of degree n such that Simpson's $\frac{1}{3}$ integrates $P_n(x)$ exactly. (Consider the interval of integration is [0, 1] for simplicity). 3+2

6. Given a table of function $f(x)$ for the points $x_r = x_0 + rh (h > 0,\ r = 0,\ 1,\ 2,...,n)$. Obtain a numerical differentiation formula for computing $f'(x_0)$, assuming appropriate interpolation formula. 'Large errors are often found in approximating derivatives from the polynomial interpolation formulae'— discuss the main reasons behind it. 3+2

7. Calculate the approximate values of (i) $\left[\frac{dy}{dx}\right]_{x=2.8}$ and (ii) $\left[\frac{d^2y}{dx^2}\right]_{x=3}$ using the following table :

x	2.0	2.2	2.4	2.6	2.8	3.0
y	0.6020	0.6848	0.7604	0.8299	0.8943	0.9542

3+2

Unit - 4

Answer ***any two*** questions.

8. Explain the method of fixed point iteration for approximating a real root of the equation of the form $x = \varphi(x)$ where $\varphi(x)$ and $\varphi'(x)$ are continuous on an interval containing the required root. Derive a sufficient condition of convergence of the above method. 3+2

9. Find the root of the equation $xe^x = \cos(x)$ using the Secant method correct to three decimal places. 5

10. Define order of convergence of an iterative method. Deduce that order of convergence of Newton-Raphson method is 2. Given that $f(x) = x - 2 + \ln(x)$ has a root near $x = 1.5$. Use Newton-Raphson method to get a better estimate. 1+2+2

11. Describe Newton's method for solving a system of equations

$$f(x, y) = 0$$
$$g(x, y) = 0$$

in two variables x and y. When does the method fail? 4+1

Unit - 5

Answer ***any two*** questions.

12. (a) What do you mean by the partial pivoting in solving of system of n linear equations in n unknowns? What are the reasons for such pivoting?

(b) Compute the total number of arithmetic operations (multiplication/division) in Gaussian algorithm for solving an $(n \times n)$ system of linear equations. 2+1+2

13. Show that for solving a system of linear equations by a suitable iterative method, a sufficient condition is that the system should be diagonally dominant. Is it possible that some system which is not diagonally dominant may converge to its solution by the iterative method? Justify. 3+2

14. Obtain the LU factorization of the matrix $A = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & -2 \\ 3 & 2 & -4 \end{bmatrix}$. Use this factorization to solve the system $AX = \begin{bmatrix} 4 \\ 4 \\ 6 \end{bmatrix}$. 5

15. Describe the power method to calculate the numerically greatest eigenvalue of a real non-singular square matrix of order n. How do you find its numerically least eigenvalue? 4+1

Unit - 6

Answer ***any one*** question.

16. Find the approximate solutions by Picard's iteration method to the initial value problem $\frac{dy}{dx} = 1 + y^2$ with the initial condition $y(0) = 0$. Hence find the approximate value of y at $x = 0.1$ and $x = 0.2$. 5

17. Find the value of $y(0.4)$ using Runge-Kutta method of fourth-order with $h = 0.2$ given that

$$\frac{dy}{dx} = \sqrt{x + y^2},\ y(0) = 0.4$$

correct up to four decimal places. 5