## 2020

## **MATHEMATICS** — **HONOURS**

Paper: CC-7

Full Marks: 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

R denotes the set of real number

Group - A

(Marks: 20)

1.	Answer the following multiple choice questions with only one correct optic	n. Choose the correct option
	and justify;	$(1+1)\times 10$

(a)	If $x, x^2, x^3$	are thr	ree linearly	independent	solutions	of a	third-order	differential	equation,	then	the
	Wronskian	W of t	the function	ns has value							

- (i)  $W = 2x^3$  (ii)  $W = x^3$  (iii)  $W = x^2$  (iv)  $W = 2x^2$ .
- (b) One of the points which lies on the solution curve of the differential equation (y-x)dx + (x+y)dy = 0 with given condition y(0) = 1 is
- (i) (1,-2) (ii) (2,-1) (iii) (2,1) (iv) (-1,2). (c) If the integrating factor of  $(x^7y^2 + 3y)dx + (3x^8y - x)dy = 0$  is  $x^my^n$ , then
  - (i) m = -7, n = 1 (ii) m = 1, n = -7 (iii) m = 0, n = 0 (iv) m = 1, n = 1.
- (d) Let y(x) be the solution of the differential equation  $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$ , y(0) = 1,  $\frac{dy}{dx}\Big|_{x=0} = -1$ , then y(x) attains its maximum value at
  - (i)  $ln\frac{4}{3}$  (ii)  $ln\frac{3}{4}$  (iii)  $ln\frac{1}{2}$  (iv) none of these.
- (e) Consider the differential equation  $a\frac{dy}{dx} + by = ce^{-\lambda x}$ , where a, b, c are positive constants and  $\lambda$  is a non-negative constant. Then every solution of the differential equation approaches to  $\frac{c}{b}$  as  $x \to +\infty$  when
  - (i)  $\lambda > 0$  (ii)  $\lambda = 0$  (iii)  $\lambda = \frac{b}{a}$  (iv)  $\lambda = \frac{a}{b}$ .

(f) Which one of the following is correct for the linear differential equation

$$(x^2 + x)\frac{d^2y}{dx^2} - 2(x+1)\frac{dy}{dx} + 2y = 0?$$

- (i) 0 is an ordinary point
- (ii) −1 is a regular singular point
- (iii) -1 is an irregular singular point
- (iv) 0 is an irregular singular point.
- (g) The initial value problem  $x \frac{dy}{dx} = y$ , y(0) = 0,  $x \ge 0$  has
  - (i) no solution

- (ii) a unique solution
- (iii) exactly two solutions
- (iv) uncountably many solutions.
- (h) The double limit  $\lim_{(x,y)\to(0,0)} \frac{|x|}{y^2} e^{-\frac{|x|}{y^2}}$ 
  - (i) does not exist

- (ii) exist and equal to 0
- (iii) exist and equal to 1
- (iv) exist and equal to -1.
- (i) Consider the function  $f(x, y) = x^2 4xy + 4y^2 + 2x^4 + 3y^4$ , then
  - (i) f has no extrema at (0, 0)
  - (ii) f has maximum value at (0, 0) which is 0
  - (iii) f has maximum value at (0, 0) which is 1
  - (iv) f has minimum value at (0, 0) which is 0.
- (j) Let  $T(x, y, z) = xy^2 + 2z x^2z^2$  be the temperature at the point (x, y, z). The unit vector in the direction in which the temperature decreases most rapidly at (1, 0, -1) is

$$(i) \quad -\frac{1}{\sqrt{5}}\hat{i} + \frac{2}{\sqrt{5}}\hat{k}$$

(ii) 
$$\frac{1}{\sqrt{5}}\hat{i} - \frac{2}{\sqrt{5}}\hat{k}$$

(iii) 
$$\frac{2}{\sqrt{14}}\hat{i} + \frac{3}{\sqrt{14}}\hat{j} + \frac{1}{\sqrt{14}}\hat{k}$$

(iv) 
$$-\frac{1}{\sqrt{5}}\hat{i} - \frac{2}{\sqrt{5}}\hat{k}$$
.

Group - B

(Marks:30)

Answer any six questions.

5×6

2. Show that a constant K can be found so that  $(x+y)^K$  is an integrating factor of

$$(4x^2 + 2xy + 6y)dx + (2x^2 + 9y + 3x)dy = 0$$

and hence solve the equation.

- 3. Reduce the equation  $x^3p^2 + x^2yp + a^3 = 0$  to Clairaut's form by the substitution y = u and  $x = \frac{1}{v}$  and obtain the complete primitive.
- 4. Solve using the method of undetermined coefficients, the equation with initial conditions,

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^x - 10\sin x, \quad y(0) = 2 \text{ and } y'(0) = 4.$$

- 5. Solve by the method of variation of parameters the equation  $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} \frac{1}{x^2}y = \log x, (x > 0).$
- 6. Solve  $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos\log(1+x)$  by changing the independent variables.
- 7. Use D-operator to solve:

$$\frac{d^2y}{dx^2} - y = x\sin x + \left(1 + x^2\right)e^x$$

- 8. Show that the equation of the curve, whose slope at any point (x, y) is equal to  $xy(x^2y^2 1)$  and which passes through the point (0, 1) is  $x^2y^2 = 1 y^2$ .
- **9.** Solve for x from the system of equations

$$\frac{dx}{dt} + 4x + 3y = t$$

$$\frac{dy}{dt} = -\frac{1}{2}$$

$$\frac{dy}{dt} + 2x + 5y = e^t$$

10. Consider the plane autonomous system

$$\frac{dx}{dt} = 2x + y, \quad \frac{dy}{dt} = 3x + 4y$$

Find the general solution of the system. State the nature of the critical point of the system. Discuss its stability. Draw a phase portrait of the system.

11. Solve the equation  $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 + 2)y = 0$  in series about the ordinary point x = 0.

(4)

## Group - C

(Marks: 15)

Answer any three questions.

- 12. Define limit point of a subset of  $\mathbb{R} \times \mathbb{R}$ . If  $B = \{(a, 0); a \in \mathbb{R}\}$ . Show that B is a closed set but not open in  $\mathbb{R} \times \mathbb{R}$ .
- 13. State the sufficient conditions for differentiability of a function  $f: \mathbb{R}^2 \to \mathbb{R}$ . Examine whether the sufficient conditions of differentiability are satisfied for the following function f(x, y) and hence comment

on differentiability of 
$$f(x, y)$$
 at  $(0, 0)$  where  $f(x, y) = \begin{cases} xy \frac{x^2 - y^2}{x^2 + y^2}, & \text{when } x^2 + y^2 \neq 0 \\ 0, & \text{when } x^2 + y^2 = 0. \end{cases}$  1+4

14. If z is a function of two variables x and y and  $x = c \cosh u \cos v$ ,  $y = c \sinh u \sin v$  (c is a real number), show that

$$\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} = \frac{c^2}{2} \left( \cosh 2u - \cos 2v \right) \left( \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} \right).$$

- 15. Find all critical points of the function  $f: \mathbb{R}^2 \to \mathbb{R}$  given by  $f(x, y) = x^3 + y^3 3x 12y + 40$  for  $(x, y) \in \mathbb{R}^2$ . Also examine whether the function f attains a local maximum or a local minimum at each of these critical points.
- 16. Find the volume of the greatest rectangular parallelopiped that can be inscribed in the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$
, by the method of Lagrange's multipliers.