2020

MATHEMATICS — GENERAL

Paper : DSE-A-2

(Graph Theory)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Day 2

1. Choose the correct alternatives : 1×10

(a) A graph consists of

(i) Vertices (ii) Edges

(iii) Both vertices and Edges (iv) None of these.

(b) Pendant vertex is a vertex with degree

(i) 0 (ii) 1 (iii) 2 (iv) None of these.

(c) The number of edges in a complete graph with n vertices is

(i) n (ii) $n\,(n + 1)/2$ (iii) $n(n - 1)/2$ (iv) None of these.

(d) The maximum number of edges in a bipartite graph having 9 vertices is

(i) 20 (ii) 18 (iii) 14 (iv) None of these.

(e) Two graphs are isomorphic to one another if there is

(i) One-one correspondence between their vertices

(ii) One-one correspondence between their edges

(iii) Both (i) and (ii)

(iv) None of these.

(f) A graph with all vertices having equal degree is known as a

(i) Multi graph (ii) Regular graph

(iii) Simple graph (iv) Complete graph.

(g) An adjacency matrix $X = (x_{ij})$ of a graph G with n vertices and no parallel edges is

(i) an n by n symmetric matrix

(ii) a binary matrix

(iii) a matrix where $x_{ij} = 0$ if there is no edge between ith and jth vertex

(iv) a matrix with all of these properties.

(h) If the starting and ending vertices of a walk are same then the walk is known as a

(i) open walk (ii) path (iii) closed walk (iv) circuit.

(i) A tree with n vertices has

(i) n edges (ii) $n+1$ edges (iii) $n-1$ edges (iv) none of these.

(j) Every tree has

(i) one centre (ii) two centres

(iii) one or two centres (iv) neither one nor two centres.

2. Answer ***any three*** questions :

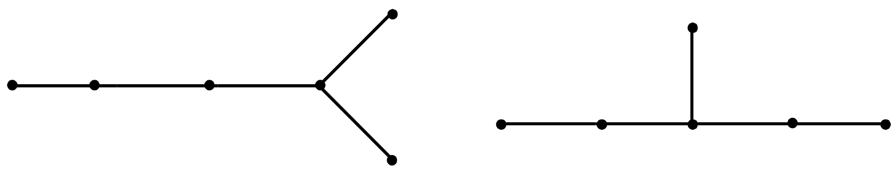
(a) (i) Define a 'graph' and 'degree of a vertex' in a graph.

(ii) What is a complete graph. Give an example. (2+1)+(1+1)

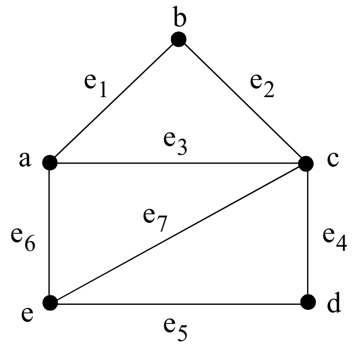
(b) Prove that the number of odd vertices in a simple graph is always even. 5

(c) (i) When are two graphs called isomorphic?

(ii) Show that the following graphs are not isomorphic. 2+3



(d) Write the adjacency matrix of the following graph : 5



(e) Define a tree. Prove that a graph is a tree if it is minimally connected. 1+4

3. Answer ***any four*** questions :

(a) (i) Define with an example for each :

(I) Simple graph, (II) Multi graph, (III) Weighted graph.

(ii) How many nodes are necessary to construct a graph with exactly 6 edges in which each node is of degree 2? (2+2+2)+4

(b) (i) Find the maximum number of edges in a simple graph with n vertices.

(ii) Write short notes on 'selfloop', 'parallel edges', 'regular graph'. 4+(2+2+2)

(c) (i) What are 'bipartite graphs' and 'complete bipartite graphs'. Explain with an example for each.

(ii) Define Kuratowski's two graphs and draw each of them. Are these graphs bipartite graphs? Justify your answer. 4+6

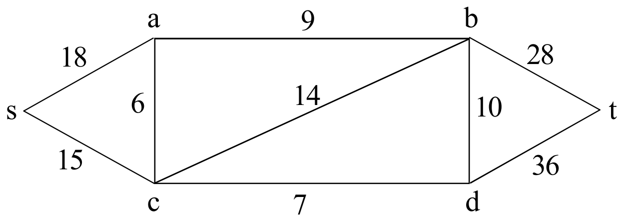
(d) (i) Define a planar graph with an example.

(ii) State and prove Euler's formula for planar graphs.

(iii) Use this formula to check whether the complete graph K_5 is planar or not. 2+5+3

(e) (i) Write Dijkstra's algorithm for finding the shortest path in weighted graphs.

(ii) Using Dijkstra's algorithm, find the shortest path between the vertices s and t in the following graph : 4+6



(f) (i) Define path and circuit. Give an example for each.

(ii) What are Eulerian circuits and Hamiltonian circuits?

(iii) Draw the graph corresponding to the following adjacency matrix : 4+3+3

$$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

(g) (i) State the travelling salesman problem.

(ii) Represent the problem with a suitable graph for 5 cities.

(iii) Is the graph a weighted graph? If so, then what do the weights represents?

(iv) What condition will make the graph a complete graph?

(v) How can the problem be solved from its graph? 2+3+2+1+2