2020

PHYSICS — HONOURS

Paper : CC-5

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any four*** from the rest.

1. Answer ***any five*** questions : 2×5

(a) Show that any function $f(x+t)$ is a solution of the equation

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial t},$$

when $u = u(x, t)$.

(b) Using Gamma function integral, prove that $0! = 1$.

(c) Using generating function for Legendre Polynomial, prove that $P_{2l+1}(0) = 0$.

(d) Find the indicial equation for the differential equation $\left(1-x^2\right)\frac{d^2y}{dx^2} + px\frac{dy}{dx} + qy = 0$, where p and q are constants.

(e) Find the non zero Fourier coefficients of

$f(x) = \cos^2 x, \quad 0 \le x < 2\pi.$

(f) Find the Fourier transform of the Dirac δ-function, $\delta(x-a)$.

Or,

For a generalised coordinate q, the Lagrangian is given by

$L = \alpha\dot{q}^2 + \beta q\dot{q} - \gamma \sin q$,

where α, β, γ are constants. Find the Hamiltonian H.

(g) For a random variable X, one finds $\langle X \rangle = 2$ and $\langle X^2 \rangle = 8$. Find the standard deviation σ_x.

Or,

For the Hamiltonian

$H = ap^2 + bx^2 - cxp$

(where a, b, c are constants), find the Hamilton's equation of motion.

2. (a) Find the probability density function $f(x)$ for the position of a particle executing SHM on $(-a, a)$ along the x-axis.

(b) Let X be a random variable having a normalized density function

$$f(x) = \begin{cases} Ce^{-x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Find :

(i) the value of $|C|$ where C is a constant.

(ii) the variance σ^2.

(iii) the probability $P(1 \leq x \leq 2)$. 3+(2+3+2)

Or,

(a) Write Lagrange's equation in cylindrical co-ordinates for a particle of mass m, moving in the gravitational potential $V = mgz$ starting from the Lagrangian.

(b) Is there any cyclic co-ordinate? If yes, find the corresponding conserved quantity.

(c) Given two points P_1 and P_2 (not too far apart), we draw a curve joining them and revolve it about the x-axis. Find the curve for which the surface area is minimum. 3+2+5

3. (a) Consider a cycloid with parametric equation $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$. Show that the time for a particle to slide without friction along the curve from (x_1, y_1) to origin is independent of starting point. (use beta function to evaluate the integral).

(b) Prove $B(m, n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$, symbols are usual. 5+5

4. (a) Given $f(x) = \begin{cases} 0, & 0 < x < l \\ 1, & l < x < 2l \end{cases}$.

Expand the function in Fourier series of period $2l$.

(b) Let a function $f(x)$ be expanded in Fourier series. Show that average of $[f(x)]^2$ over a period $= \left(\dfrac{a_o}{2}\right)^2 + \dfrac{1}{2}\sum_{n=1}^{\infty} a_n^2 + b_n^2$.

(c) Using the result of (b), show that using $f(x) = x$, $-1 < x < 1$ the infinite sum $\sum_{1}^{\infty} \dfrac{1}{n^2} = \pi^2/6$.

4+3+3

5. (a) Use Rodrigue's formula

$$P_l(x) = \frac{1}{2^l \, l!} \frac{d^l}{dx^l} \left(x^2 - 1\right)^l .$$

Show that $P_l(1) = 1$.

(b) Using (a), find $P_3(x)$. Plot $P_3(x)$ as a function of x where $-1 \le x \le 1$.

(c) Calculate $\int_{-1}^{+1} x^2 P_3(x) dx$. 3+(2+2)+3

6. Consider the differential equation $\frac{d^2 y(x)}{dx^2} + y(x) = 0$

(a) Check whether $x = 0$ is an ordinary point or a regular singular point.

(b) Find the indicial equation.

(c) From the indicial equation, find two linearly independent solutions of the given differential equation. 2+2+(3+3)

7. (a) Consider the one-dimensional wave equation for waves propagating along a string of length l. Its ends are fixed at $x = 0$ and $x = l$. The string is struck by a fine hammer such that it has an initial displacement zero everywhere but has an initial velocity v at $x = \frac{3l}{4}$.

Find the solution of the wave equation in this case.

(b) Consider

$$\nabla^2 \phi(r, \theta, \phi) = f(r) \cdot$$

Using separation of variables, write down three ordinary differential equations. Solve the equation for ϕ coordinate. 6+3+1