2023

MATHEMATICS — GENERAL

Paper : GE/CC-4

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

The symbols and notations have their usual meanings.

Group - A

1. Choose the correct alternative : 1×10

(a) For what real values of 'α' does the set $S = \{(\alpha, 0, 1), (1, \alpha + 1, 1), (1, 1, 1)\}$ of $\mathbb{R}^3$ is linearly independent?

(i) $\mathbb{R} - \{-1, 1\}$ (ii) $\mathbb{R} - \{-1, 0\}$

(iii) $\mathbb{R} - \{0, 1\}$ (iv) $\mathbb{R} - \{-1, 0, 1\}$.

(b) In a Ring $(R, \circ, *)$, where $x \circ y = x + y - 1$, $x * y = x + y - xy$, $\forall x, y \in R$, the zero element is

(i) -1 (ii) 0

(iii) 2 (iv) 1.

(c) The eigenvalues of the square matrix $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 5 \end{pmatrix}$ are

(i) all real (ii) all purely imaginary

(iii) all purely imaginary or zero (iv) either purely real or purely imaginary.

(d) If for two events A and B, $P(A) = \frac{1}{2}$; $P(B) = \frac{2}{5}$; $P(AB) = \frac{3}{10}$, then $P(\bar{A} \mid B)$ is

(i) $\frac{1}{3}$ (ii) $\frac{2}{3}$

(iii) $\frac{1}{2}$ (iv) $\frac{1}{4}$.

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(e) For what value of 'β' the function $f(x)$ defined by

$$f(x)=\begin{cases}\beta x(1-x), & 0<x<1\\ 0, & \text{elsewhere}\end{cases}$$

is a probability density function?

(i) 6 (ii) 5

(iii) 2 (iv) 4.

(f) If X and Y are two random variables such that $\text{Var}\,X = 16$, $\text{Var}\,Y = 9$ and the correlation coefficient is $3/4$, then value of $\text{Cov}(X, Y)$ is

(i) 9 (ii) 16

(iii) 12 (iv) 24.

(g) A random variable X has probability density function

$$f(x)=\begin{cases}\frac{1}{6}, & -3<x<3\\ 0, & \text{elsewhere}\end{cases}$$

Then $P(X>1)$ is given by

(i) $\frac{1}{3}$ (ii) $\frac{1}{2}$

(iii) $\frac{1}{6}$ (iv) $\frac{3}{4}$.

(h) The value of the FORTRAN expression : $J+L*I/3+J/I-K**L/I$, where $I=4$, $J=5$, $K=3$ and $L=2$ is

(i) 5 (ii) 6

(iii) 7 (iv) −6.

(i) FORTRAN expression of $\log_e\sqrt{\frac{a}{bc}}+\log_{10}|a|$ is

(i) ALOG(SQRT(A/(B * C))) + ALOG10(ABS(A))

(ii) ALOG10(SQRT(A/(B * C))) + ALOG(ABS(A))

(iii) LOG(SQRT(A/(B * C))) + LOG10(ABS(A))

(iv) LOGE (SQRT(A/(B * C))) + ALOG10(ABS(A)).

(j) Octal number corresponding to the binary number $(1101.10111)_2$ is

(i) $(15.52)_8$ (ii) $(14.56)_8$

(iii) $(15.56)_8$ (iv) $(15.46)_8$.

Group - B

Unit - 1

(Algebra - II)

2. Answer *any three* questions :

(a) Let $M=\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix}: a,b,c,d \in \mathbb{R} \text{ and } ad-bc=1\right\}$. Show that M forms a group under matrix multiplication. 5

(b) Define a subring of a ring $(R, +, \cdot)$. Check whether $S=\{2n : n \in \mathbb{Z}\}$ is a subring of the ring $(\mathbb{Z}, +, \cdot)$ of integers. 5

(c) Let $W=\{(x, y, z) \in \mathbb{R}^3 : x-4y+3z=0\}$. Show that W is a subspace of $\mathbb{R}^3$. Find the basis and dimension of W. 3+1+1

(d) Examine whether the real quadratic form $5x^2+y^2+3z^2+4xy-2yz-2zx$ is positive definite or not. 5

(e) Verify Cayley-Hamilton's theorem for the following matrix A given by $A=\begin{pmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{pmatrix}$. If possible find A^{-1} from the result. 3+2

Unit - 2

(Computer Science and Programming)

3. Answer *any four* questions :

(a) Evaluate the octal arithmetic $(576)_8+(116)_8-(477)_8$. 5

(b) Write an algorithm to find HCF and LCM of two distinct positive integers 'm' and 'n'. 5

(c) Write a flowchart for finding the maximum of n given real numbers. 5

(d) (i) Write the full form of the terms of BIT and ASCII.

(ii) Write three differences between high level language and assembly language. 2+3

(e) (i) Write the FORTRAN expression of $\sqrt{x^3+y^2}\tan(y/x)+\frac{7}{3\sqrt{2}}\log_e z$.

(ii) Let $A=4.7$, $B=5.6$ and $M=ABS\ (A-4.0 * B)/7$. Find what value of M will be stored. 3+2

(f) Write a FORTRAN program to find the sum of squares of first 10 natural numbers. 5

(g) Write a FORTRAN program to find the area, perimeter and diagonal of a rectangle whose two adjacent sides are x and y. 5

Unit - 3

(Probability and Statistics)

4. Answer ***any four*** questions :

(a) There are two urns A and B. The urn A contains 3 white and 4 red balls while the urn B contains 4 white and 3 red balls. One ball is transferred from urn A to urn B and a ball is drawn from urn B. What is the probability that the ball is red? 5

(b) Find the mean and variance of a Binomial distribution with parameters n and p. 5

(c) There are 4 red and 6 blue balls in an urn. A man draws 2 balls at a time at random. He will get ₹ 15.00 if the balls are of same colour and he will pay ₹ 10.00 if the balls are of different colours. Find the expected value of the money which the man will receive. 5

(d) Construct a frequency distribution table with class intervals $50-69$, $70-89$, $90-109$, $110-119$, ... from the following data :

95, 131, 53, 117, 155, 100, 65, 122, 71, 78, 153, 90, 125, 80, 105, 137, 65, 136, 145, 120.

Also draw the histogram for the above distribution in plane paper. 3+2

(e) If two regression lines involving two variables x and y are $y = 5.6 + 1.2x$ and $x = 12.5 + 0.6y$. Find the mean of x, y and their correlation coefficient. 5

(f) Define an unbiased and consistent estimate of a parameter in population distribution. For a random sample of size n for any (m, σ) population, prove that the sample mean is an unbiased estimate of population mean. 1+1+3

(g) Find a 95% confidence interval for the mean of a normal distribution with $\sigma = 3$, given the sample $(2.3, -0.2, -0.4, -0.9)$. Given $P(U \geq 1.96) = 0.025$, where U is $N(0, 1)$ variate. 5