2023

MATHEMATICS — HONOURS

Paper : DSE-A-2.3

(Fluid Statics and Elementary Fluid Dynamics)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Symbols have their usual meanings.

1. Answer all questions with proper explanation/justification (one mark for correct answer and one mark for justification). 2×10

(a) The dimension of viscosity is

(i) $M^2L^2T^{-1}$ (ii) $ML^{-1}T^{-1}$

(iii) MLT^{-1} (iv) L^2T^{-1}.

(b) A floating body is said to be in a state of stable equilibrium

(i) when its metacentric height is zero.

(ii) when metacentre is above the centre of gravity of the body.

(iii) when metacentre is below the centre of gravity of the body.

(iv) only when its centre of gravity is below its centre of buoyancy.

(c) The total pressure on a plane surface inclined at an angle θ with the horizontal is equal to

(i) pA (ii) $pA\sin\theta$

(iii) $pA\cos\theta$ (iv) $pA\tan\theta$

where p is the pressure at the centroid of the area and A is the area of plane surface.

(d) A triangular lamina is immersed in a heavy homogeneous liquid in such a way that its one vertex is in the effective surface and its base horizontal and at a depth 'h' from the effective surface. The depth of the centre of pressure of the lamina from the effective surface is

(i) h (ii) $\frac{h}{2}$ (iii) $\frac{3h}{2}$ (iv) $\frac{3h}{4}$.

(e) In an isothermal atmosphere, the pressure equation at a point at height z, where the gravity varies inversely as the square of the distance, is given by (with usual notation)

(i) $dp = -g\rho\, dz$ (ii) $dp = -\frac{r^2}{(r+z)^2} g\rho\, dz$

(iii) $dp = -\frac{r^2}{r+z} g\rho\, dz$ (iv) $dp = -\frac{r}{(r+z)^2} g\rho\, dz$.

(f) The depth of a lamina immersed in a heavy homogeneous liquid at rest is changed to $\frac{3}{2}$ of its original value. If C and C' be the positions of the centres of pressure in the original and changed states of the lamina respectively, then $GC : GC'$ (G being the position of c.g. of the lamina) will be

(i) $3:2$ (ii) $2:3$

(iii) $1:2$ (iv) $2:1$.

(g) The velocity vector $\vec{q}$ at a point in a fluid in motion, is given by $\vec{q} = \hat{i}x - \hat{j}y$. Then the equation of the stream lines are

(i) $xy = c$ (ii) $y^2 = 4ax$

(iii) $x^2 + y^2 = c$ (iv) $x^3 + y^3 = c$,

where c and a are constants.

(h) Which of the following potentials satisfies continuity equation?

(i) x^2y (ii) $x^2 - y^2$

(iii) $\cos x$ (iv) $x^2 + y^2$.

(i) Material derivative $\frac{D}{Dt}$ in terms of $\vec{q}$ (velocity of a fluid) is defined by

(i) $\frac{D}{Dt} \equiv \frac{\partial}{dt} - \vec{q}\cdot\vec{\nabla}$ (ii) $\frac{D}{Dt} \equiv \frac{\partial}{dt} + \vec{q}\cdot\vec{\nabla}$

(iii) $\frac{D}{Dt} \equiv \frac{\partial}{dt} + \vec{\nabla}\cdot\vec{q}$ (iv) $\frac{D}{Dt} \equiv \frac{\partial}{dt} - \vec{\nabla}\cdot\vec{q}$

(j) The Euler equation of motion is

(i) $\frac{D\vec{q}}{Dt} = \vec{F} - \frac{1}{\rho}\vec{\nabla}p$ (ii) $\frac{D\vec{q}}{Dt} = \vec{F} + \frac{1}{\rho}\vec{\nabla}p$

(iii) $\frac{D\vec{q}}{Dt} = \vec{F} - \rho\vec{\nabla}p$ (iv) $\frac{D\vec{q}}{Dt} = \vec{F} + \rho\vec{\nabla}p$

Unit - 1

2. Answer ***any one*** question :

(a) (i) Define compressibility of a fluid and an incompressible flow.

(ii) A gas satisfying Boyle's law $p = k\rho$ is acted on by force having components

$$X = -\frac{y}{x^2 + y^2}, Y = \frac{x}{x^2 + y^2}.$$

Show that the density varies as $e^{\frac{\theta}{k}}$, where $\tan\theta = \frac{y}{x}$. 2+3

(b) Prove that in a fluid under conservative field of force, the surfaces of equipressure, equidensity and equipotential energy coincide. 5

Unit - 2

3. Answer *any two* questions :

(a) (i) A hollow weightless cone of vertical angle 2α is filled with fluid, and suspended freely from a point on the rim of its base. Show that the thrust on the base is to the weight of fluid the cone would contain in the ratio $12\sin^2\alpha$ to $\cos\alpha\cdot\sqrt{1+15\sin^2\alpha}$.

(ii) Prove that the centre of pressure of a completely submerged plane surface in a fluid under the action of gravity is always below the centre of gravity of the submerged surface. 5+5

(b) (i) A lamina in the form of a parallelogram is immersed vertically in a liquid with a corner in the surface. If a and b be the depths of the adjacent corners, prove that the depth of the centre of pressure of the lamina is $\dfrac{2a^2+3ab+2b^2}{3(a+b)}$.

(ii) A spherical shell formed of two halves in contact along a vertical plane is filled with water. Prove that the resultant thrust on either half of the shell is $\dfrac{1}{4}\sqrt{13}$ of the total weight of the water. 5+5

(c) (i) A body floats freely in a heavy homogeneous liquid at rest and is given a small rotational displacement about a line of symmetry of the surface section in such a way that the quantity of the displaced liquid remains unaltered and the metacentre exists. Prove, with usual notation that $HM = \dfrac{AK^2}{V}$.

(ii) A cubical solid floats in stable equilibrium in a fluid with two of its faces horizontal. Prove that the specific gravity of the cube relative to the liquid can not lie between $\dfrac{3-\sqrt{3}}{6}$ and $\dfrac{3+\sqrt{3}}{6}$. 6+4

(d) (i) Establish the relation $p = R\rho T$, among the pressure, density and absolute temperature in an atmosphere, the symbols having their usual meanings.

(ii) A gaseous atmosphere in equilibrium is such that $p = K\rho^{\gamma} = R\rho T$, where p, ρ, T are respectively the pressure, density and temperature and K, γ, R are constants. Prove that the temperature decreases upwards at a constant rate given by $\dfrac{dT}{dz} = -\dfrac{g}{\rho}\cdot\dfrac{\gamma-1}{\gamma}$. 5+5

Unit - 3

4. Answer *any one* question :

(a) (i) Explain the difference between Lagrangian method of description and Eulerian method of description of fluid motion.

(ii) Establish the relation between Eulerian and Lagrangian description of fluid flow.

(iii) For a two-dimensional flow, the velocity components at a point in a fluid may be expressed in the Eulerian coordinates by $u = x + y + 2t$ and $v = 2y + t$.

Determine the Lagrangian coordinates as function of the initial positions x_0 and y_0 and the time t. 2+3+5

(b) (i) What is meant by the material derivative of a fluid property? Mention the two parts of the material derivative of the fluid property.

(ii) The velocity field at a point in a fluid is given by $\vec{q} = \left(\frac{x}{1+t}\right)\hat{i} + \left(\frac{y}{1+t}\right)\hat{j} + \left(\frac{z}{1+t}\right)\hat{k}$.

Determine the streamlines and the path of the particle. 5+5

Unit - 4

5. Answer *any two* questions :

(a) Derive the continuity equation of a compressible flow of a fluid in a rectangular cartesian system and hence find it for a steady incompressible flow. 5

(b) For a flow in the xy-plane, the y-component of velocity is given by $v = y^2 - 2x + 2y$.

Determine a possible x-component for a steady incompressible flow. How many possible x-components are there? 5

(c) In terms of cartesian coordinates x, y, z prove that the continuity equation is

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0,$$

where ρ is the density and v_x, v_y, v_z are the velocity components. 5

(d) State Reynold's Transport theorem. Explain how the Reynold's transport theorem is applied to conservation of momentum of a control volume system. 5
