2022

PHYSICS — HONOURS

Paper : DSE-B2

[(a) Communication Electronics]

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer *any five* questions : 2×5

(a) In an FM system, the frequency deviation is about 20·5 kHz and a modulating signal frequency is 5 kHz. Determine the modulation index and carrier swing.

(b) Assume the message signal $x(t) = 15 \cos(2\pi t)$ volts and carrier wave $c(t) = 45 \cos(100\pi t)$ volts. Derive AM wave for 30% modulation.

(c) What is sampling? State sampling theorem.

(d) What is multiplexing? Name the types of multiplexing.

(e) What is demodulation?

(f) Explain the following terms in connection with satellite communication :

(i) geostationary satellite,

(ii) uplink and downlink frequencies.

(g) What are SIM and IMEI number in mobile communication?

Group - B

2. Answer *any three* questions :

(a) (i) Show that the total power of a fully amplitude modulated wave is 1·5 times the unmodulated carrier power.

(ii) Show that the AM wave can be represented by a carrier and two side bands. 3+2

(b) How can you design an amplitude modulator by using an amplifier whose input v_i and output (v_o) characteristics is $v_o = a_1 v_i + a_2 v_i^2$? where a_1 and a_2 are constants. 5

(c) (i) How is digital modulation different from analog modulation?

(ii) Describe amplitude shift keying (ASK).

(iii) Define bit rate. 2+2+1

(d) (i) Find the Nyquist rate for the signal $x(t) = 25 \cos (500\pi t)$.

(ii) Find the bandwidth of 8-PSK.

(iii) The upper and lower cut-off frequencies of a resonant circuit are found to be 8·07 MHz and 7·93 MHz respectively. Calculate the bandwidth. 2+2+1

(e) What do you mean by transponder in satellite communication? What are their basic components? 3+2

Group - C

Answer ***any four*** questions.

3. An audio signal : $15 \sin 2\pi(1500t)$

Amplitude modulates a carrier : $60 \sin 2\pi(1000000t)$.

(a) Sketch the audio signal.

(b) Sketch the carrier.

(c) Construct the modulated wave.

(d) Determine the modulation factor and percentage modulation.

(e) What are the frequencies of the audio signal and carrier?

(f) What frequencies would show up in a spectrum analysis of the modulated wave? 1+1+2+2+2+2

4. (a) Find the expression of frequency modulated (FM) wave.

(b) A 80 MHz carrier is frequency modulated, the modulation index being 4. The frequency of information signal is 10 kHz. What is the maximum frequency deviation?

(c) What do you mean by resistor noise? Calculate the thermal noise voltage developed in a resistor $R = 100\ \Omega$. The bandwidth of the circuit is 5 kHz at room temperature 30°C.

(Given $k_B = 1{\cdot}38 \times 10^{-23}$ J/K) 3+3+(2+2)

5. (a) Draw the circuit diagram for generation of PAM signal and explain its operation.

(b) Draw the block diagram of PAM signal reception.

(c) Draw the circuit diagram of a zero order holding circuit. (3+3)+2+2

6. (a) Define μ-law for companding. Define unipolar RZ and NRZ.

(b) What is constellation diagram? Draw the diagram for 8–PSK.

(c) How can non-uniform quantization be used to increase SNR? (2+2)+(2+2)+2

7. (a) What is path loss of satellite communication system? How is the path loss related to the gain and power of the transmitting and receiving antenna?

(b) In satellite communication $P_t = 23\ dB_m$, $G_t = 2dB_i$, $G_r = 2dB_i$, $P_r = -71\ dB_m$. Find the path loss. Where P_t = Power of a transmitter, G_t = Gain of a transmitter, P_r = Power of a receiver, G_t = Gain of a receives.

(c) Draw the block diagram of Earth station. (2+3)+3+2

8. (a) What is Carson's rule of thumb for the determination of bandwidth in FM station?

(b) Describe the basic principle of satellite communication.

(c) What are the differences among 2G, 3G and 4G technologies in mobile communication system? 3+4+3

Paper : DSE-B-2

[(b) Advanced Statistical Mechanics]

Full Marks : 65

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group – A

1. Answer *any five* questions : 2×5

(a) Explain the following statement — 'Negative temperatures are hotter than positive temperatures'.

(b) A three-level single particle system has five microstates with energy 0, ε, ε, ε, and 2ε. What will be the mean energy of the system if it is in equilibrium with a heat bath at temperature T?

(c) Give a schematic diagram of chemical potential (μ) vs. temperature (T) curves for two different systems — the ideal Bose gas and the ideal Fermi gas.

(d) Three identical spin 1/2 particles of mass m are free to move within a one-dimensional rigid box of length L. Assuming that they are non-interacting, find the energies of the two lowest energy eigenstates in units of $\frac{\pi^2\hbar^2}{2mL^2}$.

(e) In a thermodynamic system in equilibrium, each particle can exist in three possible states with probabilities 1/2, 1/3, and 1/6 respectively. Find the per particle entropy of the system.

(f) What is 'Chandrasekhar mass limit'?

(g) Draw the specific heat curve for a Bose gas as a function of temperature (T) on both sides of critical temperature (T_c).

Group – B

2. Answer *any three* questions : 5×3

(a) Consider a classical gas of N identical indistinguishable particles in a two-dimensional square box of side L. If the total energy of the gas is E, find the number of accessible microstates and the entropy.

(b) Calculate the pressure exerted by an ideal Fermi gas at 0 K. What is the physical reason for the non-zero pressure at absolute zero?

(c) The wave function $\psi(t)$ of an isolated system is given by $\psi = \sum_n a_n(t)\varphi_n$ where $\{\varphi_n\}$ is the complete orthonormal set of stationary wave functions. Write down the postulate of equal a priori probabilities and the random phases in terms of a_n.

(d) A particle hops on a one-dimensional lattice with lattice spacing a. The probability of the particle to the neighbouring site to its right is p, while the corresponding probability to hop to the left is q = 1−p. Find the root-mean-squared deviation $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$ in displacement after N steps.

(e) Use canonical ensemble to prove that specific heat (C_v) at thermal equilibrium cannot be negative.

Group – C

Answer ***any four*** questions.

3. (a) Show that the relative root-mean-squared fluctuation in energy of an N-particle system in contact with a heat reservoir varies as $1/\sqrt{N}$. Hence comment on the equivalence of the canonical ensemble to the microcanonical ensemble in thermodynamic limit.

(b) An ideal collection of N two-level systems is in thermal equilibrium at temperature T. Each system has a ground state energy $-\varepsilon$ and an excited state energy $+\varepsilon$. Prove that the Helmholtz free energy of the system is $A = -Nk_BT \ln\left\{2\cosh\left(\frac{\varepsilon}{k_BT}\right)\right\}$. (5+2)+3

4. (a) Write down the grand partition function of an ideal Bose gas of fugacity z, volume V and temperature T.

(b) The number of Bosons in the excited states can be expressed as $N_e = N - N_0 = \frac{V}{\lambda^3} g_{\frac{3}{2}}(z)$, where $\lambda = h/\sqrt{2\pi m k_B T}$ and $g_{\frac{3}{2}}(z)$ is the monotonically increasing Bose function. Given that the largest value (≈ 2.612) of $g_{\frac{3}{2}}(z)$ is bounded at $z = 1$, derive the condition to obtain Bose-Einstein condensate.

(c) Show that in the condensed phase ($T < T_C$), $N = N_0 + N\left(\frac{T}{T_C}\right)^{3/2}$. 2+5+3

5. Consider the ionization of atomic hydrogen into a hydrogen ion and an electron : $H \rightleftharpoons H^+ + e^-$. The number densities of the neutral hydrogen atoms, the hydrogen ions and the electrons at temperature T are n_H, n_{H^+}, and n_e respectively.

(a) Ignoring the excited bound states derive the Saha ionisation equation

$$\frac{n_H + n_e}{n_H} = \frac{g_H + g_e}{g_H} \frac{\lambda_H^3}{\lambda_{H^+}^3 \lambda_e^3} e^{-I/k_BT}$$

where the gs represent the statistical weights of the three species — the neutral hydrogen atoms, the hydrogen ions and the electrons. The λs represent their thermal wavelengths $\left(\lambda = h/\sqrt{2\pi m k_B T}\right)$ and I is the ionization energy of a hydrogen atom.

(b) Since, $g_H = g_e = 2$, $g_{H+} = 1$, $m_{H+} \approx m_H$ and $n_H = n_e$ (overall charge neutrality) show that we may write Saha's equation as $\frac{x^2}{1-x} = \frac{1}{n_e \lambda_e^3} e^{-1/k_B T}$, where $x = \frac{n_{H^+}}{(n_H + n_{H^+})}$ is the fraction of hydrogen atoms that are ionised.

(c) At the surface of the sun the temperature is about 5800 K and the number density of electrons is $2 \times 10^{19}\ m^{-3}$. Using the Saha's equation in (b) show that less than one hydrogen atom in 10000 is ionised. 5+3+2

6. (a) The energy eigenvalues of a one-dimensional harmonic oscillator are given by

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega,\ n = 0, 1, 2, \ldots$$

Find the internal energy of a system of N such independent harmonic oscillators in thermal equilibrium at temperature T. Calculate $C_P - C_V$ for this system.

(b) The Hamiltonian of a classical oscillator in two-dimension in plane polar coordinate is

$$H = \frac{p_r^{\ 2}}{2m} + \frac{p_\theta^{\ 2}}{2mr^2} + \frac{1}{2}kr^2$$

where the symbols have their usual meaning.
Calculate ‹ H ›. You may use the generalised equipartition theorem. (4+2)+4

7. (a) Consider the Hamiltonian for the Ising model with N spins

$$H = -J\sum_{\langle ij \rangle} s_i s_j$$

with $s_i = \pm 1$, $J > 0$ and $\sum_{\langle ij \rangle}$ is a sum over nearest neighbours. Within Bragg Williams approximation the average magnetization per spin (m) can be expressed by the following relation :

$$m = \tanh(J\gamma m / k_B T),$$

where γ is the number of nearest neighbours. Use this relation to show that there exists a critical temperature $T_C = J\gamma / k_B$ below which the system can have a non-zero spontaneous magnetization and above cannot.

(b) Calculate the entropy $S = -k_B T_r(\rho ln\rho)$ for the following density matrix $\rho = \begin{bmatrix} \tau - 1 & 0 \\ 0 & \tau + 1 \end{bmatrix}$, where τ is a real parameter and the rest of the symbols have usual meaning.

(c) For a system having $V^{2/3} E =$ constant, calculate the pressure, P as a function of energy, E and volume, V. Hence find the relation between P, V and E. 4+3+3

8. (a) Consider the degeneracy parameter $e^{\alpha} = e^{-\left(\frac{E_F}{kT}\right)}$ of FD gas. Now, depending on this temperature how would you classify the degenerate state of FD gas? (whether it is very weakly degenerate, weakly degenerate, degenerate or strongly degenerate?)

(b) Show that the number of Fermions per unit volume of a strongly degenerate FD gas is

$$n = \left(\frac{8\pi}{3}\right)\left(\frac{2m}{\hbar^2}\right)^{3/2} F_{F_0}^{3/2}.$$

(c) What is Fermi temperature? Show that the degeneracy parameter $p_0 = \frac{2}{5} n k_B T_F$, where n is the number of molecules per unit volume and T_F is the Fermi temperature. (2+4)+1+3