## 2024

## ADVANCED BUSINESS MATHEMATICS — HONOURS

Paper: DSE-5.1 AH

(Module - II)

Full Marks: 40

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any four questions.

1. (a) If  $f(x) = \frac{x+|x|}{x-|x|}$ , find domain and range of the function f.

(b) Evaluate: 
$$Lt \frac{2x^2 - 7x + 6}{3x^2 - 7x + 2}$$
. 5+5

2. (a) Express the following in a single matrix :  $\begin{bmatrix} 3 & -2 \\ 2 & 4 \\ 5 & -1 \end{bmatrix} \begin{bmatrix} -2 & 1 & 5 \\ 1 & -2 & 3 \end{bmatrix} - \begin{bmatrix} 8 & 2 & 3 \\ 5 & 3 & 7 \\ 9 & 11 & 2 \end{bmatrix}.$ 

(b) Prove that 
$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c).$$
 5+5

3. (a) Evaluate :  $\int \frac{dx}{(x-2)\sqrt{2x+1}}$ .

(b) If 
$$y = ae^{3x} + be^{-3x}$$
, then prove that  $\frac{d^2y}{dx^2} = 9y$ .

4. (a) Evaluate : 
$$\int_{0}^{2} \frac{x^2}{x^3 + 1} dx$$

(b) Evaluate: 
$$\int \frac{dx}{\sqrt{x+1} + \sqrt{5x+1}}$$
.

- 5. (a) Find the area above the x-axis bounded by x-2y+4=0, x=3, x=6
  - (b) A firm produces x tonnes of output at a total cost  $\mathcal{E}$ C, where  $C = \frac{1}{10}x^3 5x^2 + 10x + 5$ . At what level of output will the marginal cost attain the minimum value?
- **6.** (a) If  $f(x) = \frac{ax b}{bx a}$ , show that  $f(a)f\left(\frac{1}{a}\right) f(b)f\left(\frac{1}{b}\right) = 0$ .

(b) If 
$$y = 2x^3 + 6x^2 - 48x + 9$$
, find the values of x for which  $\frac{dy}{dx} = 0$ .

7. (a) Solve by Cramer's Rule:

$$x + y + z = 9$$
$$2x + 3y - z = 9$$
$$3x - y - z = -1$$

(b) If 
$$\begin{bmatrix} x-1 & x+1 \\ y+1 & y-1 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 5 & 3 \end{bmatrix}$$
, find x and y.

8. (a) Show that 
$$\begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = xyz(x-y)(y-z)(z-x).$$

(b) Find the matrices 
$$A$$
 and  $B$  for which  $3B - 2A = \begin{bmatrix} 10 & 1 \\ 3 & 5 \end{bmatrix}$  and  $2A + B = \begin{bmatrix} 2 & 3 \\ 5 & 1 \end{bmatrix}$ . 5+5