2023

MATHEMATICS — HONOURS

Paper : CC-14

(Numerical Methods)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Unit - 1

1. Each of the following questions has four possible answers of which exactly one is correct. Choose the correct alternative : 1×10

(a) The maximum absolute error that occurs in rounding off a number to m decimal places is

(i) $5\times10^{m-1}$ (ii) $\frac{1}{2}\times10^{-m-1}$

(iii) $5\times10^{-m-1}$ (iv) $\frac{1}{2}\times10^{-m+1}$.

(b) Find the polynomial of degree ≤ 3 passing through the points $(-1, 1)$, $(0, 1)$, $(1, 1)$ and $(2, -3)$.

(i) $\frac{1}{3}\left(-2x^3+2x+3\right)$ (ii) $-2x^3+2x^2+2x+3$

(iii) $-2x^3+2x+3$ (iv) $\frac{1}{3}\left(2x^2+2x+3\right)$.

(c) The value of $\left(\frac{\Delta^2}{E}\right)x^2$ at $h = 1$ is

(i) 3 (ii) 2

(iii) 1 (iv) 6.

(d) In the Stirling's interpolation formula, the starting point is so chosen that the value of u, where $u=\frac{x-x_0}{h}$, lies between

(i) $\frac{1}{4}<u<\frac{3}{4}$ (ii) $\frac{1}{4}<u<1$

(iii) $-\frac{1}{4}<u<\frac{1}{4}$ (iv) $-\frac{3}{4}<u<-\frac{1}{4}$.

(e) Up to which order of polynomial, Simpson's $\frac{1}{3}$rd rule provide accurate result?

(i) 2 (ii) 3

(iii) 1 (iv) None of these.

(f) To find the smallest root of the equation $x^3 = 1 - x^2$ on the interval [0, 1] by iterative method, the equation should be rewritten as

(i) $x=\sqrt{1-x^3}$ (ii) $x=\sqrt[3]{1-x^2}$

(iii) $x=\frac{1}{\sqrt{x+1}}$ (iv) $x=\frac{1-x^2}{x^2}$.

(g) The Runge-Kutta method of order four is used to solve the differential equation $\frac{dy}{dx}=f(x), y(0)=0$ with step length h. The solution at $x = h$ is given by

(i) $y(h)=\frac{h}{6}\left[f(0)+4f\left(\frac{h}{2}\right)+f(h)\right]$ (ii) $y(h)=\frac{h}{6}\left[f(0)+2f\left(\frac{h}{2}\right)+f(h)\right]$

(iii) $y(h)=\frac{h}{6}[f(0)+f(h)]$ (iv) $y(h)=\frac{h}{6}\left[2f(0)+f\left(\frac{h}{2}\right)+2f(h)\right]$.

(h) Let $A=\begin{pmatrix}1 & 1 & 2\\ 1 & 1 & 3\\ 2 & 3 & 4\end{pmatrix}, B=\begin{pmatrix}2 & 2 & 3\\ 2 & 2 & 4\\ 9 & 8 & 7\end{pmatrix}$. Consider the following two statements.

(P1) LU decomposition for the matrix A is possible.

(P2) LU decomposition for the matrix B is possible.

Which of the following statements is true?

(i) Both P1 and P2 are true (ii) Only P1 is true

(iii) Only P2 is true (iv) Neither P1 nor P2 is true.

(i) Power method is applicable on the matrix

(i) $A=\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix}$ (ii) $A=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

(iii) $A=\begin{pmatrix}1 & 1\\ 0 & 2\end{pmatrix}$ (iv) $A=\begin{pmatrix}0 & -2\\ -2 & 0\end{pmatrix}$.

(j) Which of the following is closed type quadrature ?

(i) $\int_{-1}^{1} f(x)dx = 2f(0)$ (ii) $\int_{-1}^{1} f(x)dx = \frac{1}{2}[f(0)+f(1)]$

(iii) $\int_{-1}^{1} f(x)dx = \frac{1}{2}[f(-1)+f(1)]$ (iv) $\int_{-1}^{1} f(x)dx = \frac{1}{2}[f(-1)+f(0)]$.

Unit - 2

Answer *any one* question.

2. (a) Prove that $(1 + \Delta)(1 - \nabla) = 1$.

(b) If $y = 7x^7 - 3x^3$, find the percentage error in y at $x = 1$, if the error in $x = 0.005$. 2+3

3. Derive Newton's divided difference interpolation formula for $(n + 1)$ arguments. 5

Unit - 3

Answer *any two* questions.

4. Compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 3.8$ using the following table. 5

x	0	1	2	3	4	5	6
y	31.23	32.72	33.97	34.74	35.05	34.91	34.51

5. (a) Establish the midpoint rule $\int_a^b f(x)dx = hf\left(a+\frac{h}{2}\right)+\frac{h^3}{24}f''(\xi), a \le \xi \le b, h = \frac{b-a}{2}$.

(b) Evaluate $\int_0^{10} \frac{dx}{1+x^2}$ by using Simpson's $\frac{3}{8}$ rule. 3+2

6. (a) If T_1, T_2 denote the Trapezoidal rule approximations to $I = \int_a^b f(x)dx$ with 1, 2 sub-intervals respectively, show that $(I - T_2) = \frac{1}{3}(T_2 - T_1)$.

(b) What is the degree of precision for Simpson's Three-Eight rule? 4+1

7. Derive the Weddle's rule from Newton-Cotes formula. Also, mention the degree of precision of this method. 4+1

Unit - 4

Answer ***any two*** questions.

8. Show that the square root of $N = AB$ is given by $\sqrt{N} = \frac{S}{4} + \frac{N}{S}$ where $S = A + B$. 5

9. Describe Newton's method for solving a system of equations $f(x, y) = 0$, $g(x, y) = 0$ in two variables x and y. When does the method fail? 4+1

10. (a) The equation $x^3 - 5x^2 + 4x - 3 = 0$ has one root near $x = 4$ which is to be computed by the following iterative scheme : $x_{n+1} = \frac{3 + (k-4)x_n + 5x_n^2 - x_n^3}{k}$ with $x_0 = 4$ and k as an integer. Determine the value of k that gives fastest convergence.

(b) What is / are the difference(s) between the Regula-Falsi and the Secant method. 3+2

11. Show that if the iteration function of the equation $f(x) = 0$ is such that $|g'(x)| \le k < 1$ for all x in $[a, b]$, then the sequence $\{x_n\}$ generated by $x_n = g(x_{n-1})$; $n = 1, 2, 3, \ldots$ converges to the real root of $f(x) = 0$ uniquely for any choice of x_0 in $[a, b]$. 5

Unit - 5

Answer ***any two*** questions.

12. (a) What do you mean by the partial pivoting in solving of system of n linear equations in n unknowns? What are the reasons for such pivoting?

(b) Compute the total number of arithmetic operations (multiplication / division) in Gaussian algorithm for solving an $(n \times n)$ system of linear equations. (2+1)+2

13. What is the condition of convergence of Gauss-Seidel method? Is it a necessary and sufficient condition? Compare this method with Gauss's elimination method. 2+1+2

14. Describe the power method to calculate the numerically greatest eigenvalue of a real non-singular square matrix of order n. How do you find its numerically least eigenvalue? 4+1

15. Solve the following system by Crout's method :

$$x + y + z = 3,\ 2x - y + 3z = 16,\ 3x + y - z = -3.$$ 5

Unit - 6

Answer ***any one*** question.

16. Using three successive approximations of Picard's method, obtain approximate solution of the differential equation, $\frac{dy}{dx} = x^2 + y^2$ satisfying the initial condition $y(0) = 0$. 5

17. Using Euler's modified method, solve the following differential equation

$$\frac{dy}{dx} = x^2 + y \text{ with } y(0) = 1$$

for $x = 0.02$ by taking step length $h = 0.01$. 5